CBCS SCHEME

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17MATDIP31

Third Semester B.E. Degree Examination, July/August 2022
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Find the modulus and amplitude of $1-i\sqrt{3}$ and hence express it in polar form. **(07 Marks)**

b. Express the following in the form $a+ib$ and also find the conjugate $\dfrac{1}{1-\cos\theta+i\sin\theta}$. **(07 Marks)**

c. Find the sine of the angle between $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+2\hat{k}$. **(06 Marks)**

OR

2 a. Prove that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos\dfrac{n\theta}{2}\cos\dfrac{n\theta}{2}$. **(06 Marks)**

b. Find $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$, $\vec{b}\times\left(\vec{a}\times\vec{c}\right)$ and $\vec{c}\cdot\left(\vec{a}\times\vec{b}\right)$ where $\vec{a}=\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+2\hat{k}$, $\vec{c}=3\hat{i}-\hat{j}-\hat{k}$. **(06 Marks)**

c. Find the value of λ so that the points $A(-1,4,-3)$, $B(3,2,-5)$, $C(-3,8,-5)$ and $D(-3,\lambda,1)$ may lie on one plane. **(08 Marks)**

Module-2

3 a. If $y=a\cos(\log x)+b\sin(\log x)$ prove that $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$. **(08 Marks)**

b. Find the angle between the curves $r=a\cos\theta$, $2r=a$. **(06 Marks)**

c. Using Euler's theorem, prove that $xu_x+yu_y=2\tan u$, where $u=\sin^{-1}\left(\dfrac{x^3+y^3}{x+y}\right)$. **(06 Marks)**

OR

4 a. Obtain the Maclaurin's series expansion of the function $\sqrt{1+\sin 2x}$ upto x^4. **(08 Marks)**

b. Find the pedal equation of the curve $r=a(1-\cos\theta)$. **(06 Marks)**

c. If $u=\dfrac{yz}{x}$, $v=\dfrac{zx}{y}$, $w=\dfrac{xy}{z}$ show that $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}=4$. **(06 Marks)**

Module-3

5 a. Obtain a reduction formula for $\int \sin^n x\,dx$ $(n>0)$. **(08 Marks)**

b. Evaluate $\displaystyle\int_0^\infty \frac{x^2}{(1+x^6)^{\frac{7}{2}}}dx$. **(06 Marks)**

c. Evaluate $\displaystyle\int_0^1\int_{x^2}^{x}\left(x^2+3y+2\right)dy\,dx$. **(06 Marks)**

OR

6 a. Evaluate $\int_0^1\int_0^y xy\,dx\,dy$. (08 Marks)

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax-x^2}\,dx$. (06 Marks)

c. Evaluate $\int_0^1\int_0^2\int_1^2 x^2yz\;dx\,dy\,dz$. (06 Marks)

Module-4

7 a. A particle moves along the curve $x=1-t^3$, $y=1+t^2$ and $z=2t-5$. Find the components of velocity and acceleration at $t=1$ in the direction $2i+j+2k$. (08 Marks)

b. Find the directional derivatives of $\phi=x^2yz+4xz^2$ at $(1,-2,-1)$ along $2i-j-2k$. (06 Marks)

c. Show that $\vec{F}=(y+z)i+(z+x)j+(x+y)k$ is irrotational. (06 Marks)

OR

8 a. If $\vec{F}=(x+y+z)\hat{i}+\hat{j}-(x+y)\hat{k}$, show that $\vec{F}\times\text{curl}\vec{F}=0$. (08 Marks)

b. If $\phi(x,y,z)=x^3+y^3+z^3-3xyz$, find $\nabla\phi$, $|\nabla\phi|$ at $(1,-1,2)$. (06 Marks)

c. Find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ where $\vec{F}=(xz^3\hat{i}-2x^2yz\hat{j}+2yz^4\hat{k})$ at $(1,-1,1)$. (06 Marks)

Module-5

9 a. Solve $x^2ydx-(x^3+y^3)dy=0$. (08 Marks)

b. Solve $(x^2+y)dx+(y^3+x)dy=0$. (06 Marks)

c. Solve $(5x^4+3x^2y^2-2xy^3)dx+(2x^3y-3x^2y^2-5y^4)dy=0$. (06 Marks)

OR

10 a. Solve $\frac{dy}{dx}+y\cot x=\sin x$. (08 Marks)

b. Solve $\frac{dy}{dx}-y\tan x=y^2\sec x$. (06 Marks)

c. Solve $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$. (06 Marks)

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